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Causal Set Dynamics and Elementary Particles

Alexey L. Krugly[1]

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A causal set is considered a finite, acyclic oriented graph with special restrictions: each vertex has two incident edges directed to this vertex and two incident edges directed from this vertex. This graph is called a causal graph. The vertex with incident edges is called an X-structure. Quantum measurements are discussed. A dynamics of the causal graph is a random sequence of elementary interactions of edges that is described by complex amplitudes. These amplitudes correspond to each pair of interacting edges. The edges are elementary particles. The mass of a particle is a probability of the interaction. An equation of particles is proposed. In a simple case this equation for X-structure is the Dirac's equation. The edges are fermions with the spin 1/2.

1. INTRODUCTION

A particular case of the causal set hypothesis is presented in this paper. This hypothesis asserts that a microstructure of spacetime is a causal set. The causal set is a locally finite set with a partial-order relation, namely the relation $\leqslant$ such that $\forall a, b, c \in G$,

$$a \leqslant a \text{ (reflexive relation)}, \tag{1.1}$$

$$(a \leqslant b) \wedge (b \leqslant a) \Rightarrow (a = b) \text{ (acyclic relation)}, \tag{1.2}$$

$$(a \leqslant b) \wedge (b \leqslant c) \Rightarrow (a \leqslant c) \text{ (transitive relation)}, \tag{1.3}$$

$$|[a, b]| < \infty \text{ (local finiteness)}. \tag{1.4}$$

$|[a, b]|$ is the number of elements, c, falling between a and b in the sense that $a \leqslant c \leqslant b$. The causal set hypothesis asserts that this partial-order relation is the causal relation of discrete spacetime. For a fuller introduction to causal sets see Bombelli *et al.* (1987), Reid (1999), and Sorkin (1991).

The dynamics of causal sets is not very developed. A general family of classically stochastic, sequential-growth dynamics for causal sets is studied in Rideout and Sorkin (1999, 2000) and Sorkin (2000). One of the best reasons to be interested

[1] GCMPP, Novojasenevskiy 32-1-557, 117463, Moscow, Russia; e-mail: krugly@aha.ru.

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in the classical dynamics for causal sets is that quantum gravity must possess general relativity as a classical limit. A quantum model of a causal set dynamics is studied in Criscuolo and Waelbroeck (1998). In these papers the connection between causal set and spacetime is discussed.

In my opinion the first step in developing a causal set dynamics is to construct particles instead of spacetime. Some properties of particles can be connected with some elementary interactions of a causal set. Spacetime can be a result of many such interactions. This is a more complicated thing. The proposed dynamics is based on the immediate causal priority (Finkelstein, 1988).

In the subsequent sections of this paper I describe the model, introduce the variant of a causal set dynamics, introduce the concept of a proper time and mass of particles, consider the state vectors of edges and Dirac's equation; and discuss future opportunities of the considered approach. The model in question is discussed also in Krugly (1998, 2000a,b).

2. MODEL

2.1. An Acyclic Oriented Graph

Suppose the universe consists of some pairs of discrete events that are connected by elementary discrete causal connections. Then spacetime is an oriented graph. Graph theory is presented in Ore (1962); a fragment of such a graph is represented in Fig. 1. The vertexes are world points. The oriented edges are elementary causal connections. The represented fragment contains 9 vertexes and 25 edges. The edges have arbitrary numbers. An edge is directed from a vertex

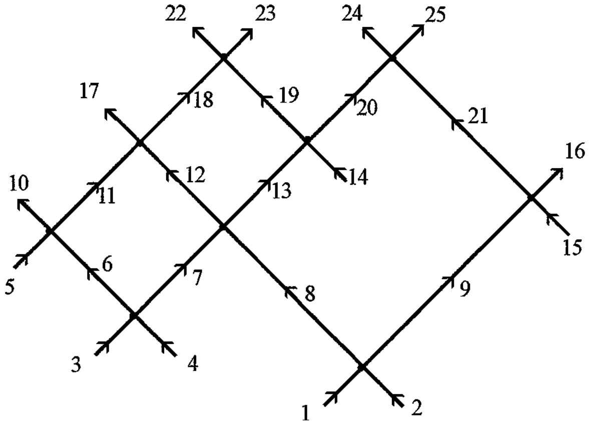

Fig. 1. An example of a graph.

cause to a vertex effect. The edges directed to a vertex are called input edges of this vertex. The edges directed from a vertex are called output edges of this vertex. In Fig. 1 the edges 6, 7, 8, 9, 11, 12, 13, 18, 19, 20, and 21 are incident to two vertexes. Such edges are called internal edges. In the graph of the universe each edge is incident to two vertexes, but in Fig. 1 some edges are linked with only one vertex because the paired vertex lies outside the figure. Such edges are called external edges. The external edges directed to a vertex of the considered fragment are called external input edges. The edges 1, 2, 3, 4, 5, 14, and 15 are the external input edges. The external edges directed from a vertex of the considered fragment are called external output edges. The edges 10, 16, 17, 22, 23, 24, and 25 are the external output edges. The graph of the universe may be finite or infinite. We do not discuss this problem. In any case we can consider only finite fragments and take into account the rest of the graph in an approximate way.

A set of edges is called a sequence if every two neighboring edges have a common vertex. A sequence is called oriented if all the edges of the sequence are included in the direction of the orientation. Two vertexes a and b have a causal connection if there is an oriented sequence between them or if these vertexes coincide. The vertexes are denoted by lowercase Latin letters. Vertex a is the cause of vertex b if vertex a is the initial vertex of this sequence. The sequence is called cyclic if the vertexes a and b coincide. The causality principle is the prohibition of oriented cyclic sequences. In this case the set of vertexes is the causal set and satisfies the conditions (1.1)–(1.4).

Suppose the graph is of the deepest level of matter. Vertexes and edges are elementary objects and have no internal structure. All the vertexes are identical and so are all the edges. All information consists in the structure of the graph. The graph makes up spacetime. Particles are parts of the graph with some symmetry.

2.2. A Duality of Vertexes and Edges

There is some symmetry between vertexes and edges. The edges can be denoted using incident vertexes. Denote by (a, b) the internal edge that is incident to the vertexes a and b. Denote by $a\uparrow$ the external output edge that is incident to the vertex a. Denote by $a\downarrow$ the external input edge that is incident to the vertex a. In these notations the external input or output edges are indiscernible if they are incident to the same vertex. These notations are called the vertex representations.

Similarly, the edges can be considered primary objects. The edges are denoted by lowercase Greek letters. Denote by $\alpha\uparrow$ the external output edge and by $\alpha\downarrow$ the external input edge. The arrows are used for the designation of output and input edges. These arrows will be omitted for simplification of the designation if possible. The vertexes can be denoted using the incident edges. Denote by $(\alpha, \beta, \gamma, \delta)$ the vertex that is incident to the edges α, β, γ, and δ. This notation is called the edge representation. The numbering of edges and vertexes is arbitrary. It is obvious that the physical meaning of any quantity should not depend on the numbering.

The causal relation of vertexes sets the causal relation of edges. Suppose a is the final vertex of the edge α and b is the initial vertex of the edge β. The edge α is the cause of the edge β if vertex a is the cause of vertex b. The causal relation is an order relation on the set of edges, namely a relation $\prec$ such that $\forall\alpha, \beta, \gamma \in G$,

$$\{\alpha \mid (\alpha \prec \alpha)\} = \varnothing \text{ (irreflexive relation)}, \tag{2.1}$$

$$\forall\alpha\ (\{\beta \mid (\alpha \prec \beta) \wedge (\beta \prec \alpha)\} = \varnothing) \text{ (acyclic relation)}, \tag{2.2}$$

$$\alpha \prec \beta \text{ and } \beta \prec \gamma \Rightarrow \alpha \prec \gamma \text{ (transitive relation)}, \tag{2.3}$$

$$|[\alpha, \beta]| < \infty \text{ (local finiteness)}. \tag{2.4}$$

The causal relation of edges is a reflexive relation if we set the edge as a cause of itself. Similarly, the causal relation of edges sets the causal relation of vertexes.

A vertex is a discrete spacetime point. An edge is an elementary timelike four-vector called chronous. It is possible that an edge is an elementary four-momentum. It is possible that the vertex representation is the discrete basis of the spacetime representation, and the edge representation is the discrete basis of the momentum representation.

2.3. A Fundamental Law of Conservation and a Binary Principle

The causality principle is not the only restriction on the structure of the graph in this paper. By assumption, the graph satisfies the following conditions:

1. The number of incident edges directed to a vertex is equal to the number of incident edges directed from this vertex. This postulate is called the fundamental law of conservation.
2. The minimum number of interacting objects is equal to two. Each vertex has two incident edges directed to this vertex. This postulate is called the binary principle. Then each vertex has two incident edges directed from this vertex. The vertex with four incident edges is called an X-structure (Fig. 2).

We can consider these two postulates to be the principles of interpretation. For example, if there is some vertex with six incident edges, we interpret it as two vertexes (Fig. 3(a)). If there is some vertex with three incident edges, we interpret it as a vertex with one more external edge (Fig. 3(b)).

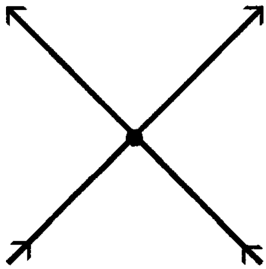

Fig. 2. The *X* structure.

A graph is called a causal graph if it is a finite acyclic oriented graph and satisfies the fundamental law of conservation and the binary principle. The graph in Fig. 1 is such a graph.

The model similar to it was called the symmetric dyadic net, and the elementary structures were called *X*-shaped pentads (in Finkelstein, 1988).

3. A SEQUENTIAL GROWTH DYNAMICS

3.1. Objects and Observers

The dynamics of causal graphs must include the influence of observers. Let us discuss relations between objects and observers. The interaction between objects and observers in quantum theory is a very complicated problem. Consider a simple example that is well known in probability theory: a shuffled pack of 52 cards.

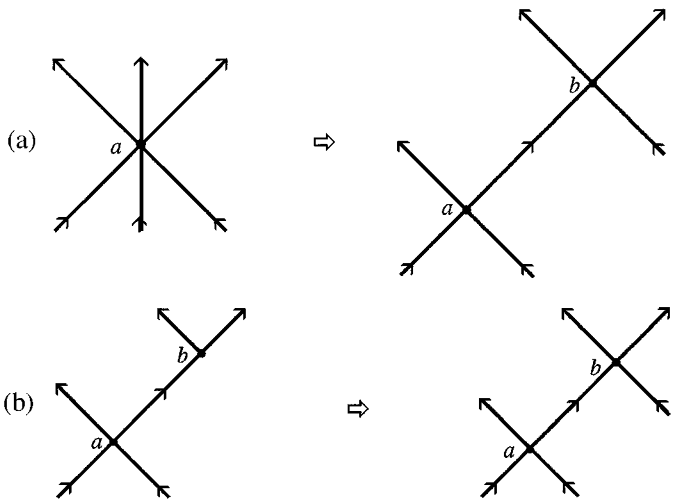

Fig. 3. The examples of the interpretation.

Assume that the observer O_1 takes the card C_1, but he does not look at this card. O_1 gets

$$\begin{aligned} P\{C_1 = 2\spadesuit\} &= \frac{1}{52} \\ P\{C_1 = 3\spadesuit\} &= \frac{1}{52} \\ &\dots \\ P\{C_1 = K\clubsuit\} &= \frac{1}{52} \\ &\dots \end{aligned} \tag{3.1}$$

for the probabilities of the denomination of the card C_1. Then the observer O_1 sends the card C_1 to a very far point of space, for example, the neighborhood of Alpha Centauri. Then the observer O_1 takes the card C_2, and looks at this card. Assume $C_2 = 2\spadesuit$. This is the obtaining of the information (the measurement). Consequently O_1 gets

$$\begin{aligned} P\{C_1 = 2\spadesuit\} &= 0 \\ P\{C_1 = 3\spadesuit\} &= \frac{1}{51} \\ &\dots \\ P\{C_1 = K\clubsuit\} &= \frac{1}{51} \\ &\dots \end{aligned} \tag{3.2}$$

for the card C_1. The probabilities of the denomination of the card C_1 at Alpha Centauri are immediately changed in the process of measurement on Earth. This effect is called the reduction of a wavepacket in quantum theory.

The considered example is more interesting if we take into account the time. Assume that the observer O_1 takes the card C_1 at the point of time T_1, but he does not look at this card. This is the first event. O_1 gets

$$\begin{aligned} P\{C_1 = 2\spadesuit, T_1\} &= \frac{1}{52} \\ P\{C_1 = 3\spadesuit, T_1\} &= \frac{1}{52} \\ &\dots \\ P\{C_1 = K\clubsuit, T_1\} &= \frac{1}{52} \\ &\dots \end{aligned} \tag{3.3}$$

for the probabilities of the denomination of the card C_1. Then the observer O_1 takes the card C_2 at the point of time $T_2 > T_1$, and he also does not look at this card. This is the second event. Then he looks at the card C_2 at the point of time $T_3 > T_2$. Assume $C_2 = 2♠$. This is the obtaining of the information (the measurement). Consequently, O_1 gets

$$\begin{gathered} P\{C_1 = 2♠, T_1\} = 0 \\ P\{C_1 = 3♠, T_1\} = \frac{1}{51} \\ \cdots \\ P\{C_1 = K♣, T_1\} = \frac{1}{51} \\ \cdots \end{gathered} \tag{3.4}$$

for the first event. The measurement at the point of time T_3 changes the probability of the event at the previous point of time T_1. This is a causal paradox if we assume that the probabilities are properties of objects. The probabilities describe the relations between objects and observers and depend on the properties of objects and the properties of observers. In theory of probability this is described as conditional probabilities.

$$\begin{gathered} P\{C_1 = 2♠, T_1 \mid O_1, T_1\} = \frac{1}{52} \\ P\{C_1 = 3♠, T_1 \mid O_1, T_1\} = \frac{1}{52} \\ \cdots \\ P\{C_1 = K♣, T_1 \mid O_1, T_1\} = \frac{1}{52} \\ \cdots \end{gathered} \tag{3.5}$$

are the probabilities of the first event for the observer O_1 at the point of time T_1.

$$\begin{gathered} P\{C_1 = 2♠, T_1 \mid O_1, T_3\} = 0 \\ P\{C_1 = 3♠, T_1 \mid O_1, T_3\} = \frac{1}{51} \\ \cdots \\ P\{C_1 = K♣, T_1 \mid O_1, T_3\} = \frac{1}{51} \\ \cdots \end{gathered} \tag{3.6}$$

are the probabilities of the first event for the observer O_1 at the point of time T_3. There are two sequences: the sequence of events of the object and the sequence of obtaining of the information. The causal order of events is independent of the causal order of obtaining of the information. An observer can obtain some information about a previous event after obtaining some information about a later event.

The probabilities of the same event can be different for different observers at the same point of time. If the observer O_2 does not look at the card C_2 at the point of time $T_3 > T_2$, he gets

$$
\begin{aligned}
&P\{C_1 = 2\spadesuit, T_1 \mid O_2, T_3\} = \frac{1}{52} \\
&P\{C_1 = 3\spadesuit, T_1 \mid O_2, T_3\} = \frac{1}{52} \\
&\cdots \\
&P\{C_1 = K\clubsuit, T_1 \mid O_2, T_3\} = \frac{1}{52} \\
&\cdots
\end{aligned}
\tag{3.7}
$$

The probabilities depend on the properties of observers that concern the relations between objects and observers. The probabilities are independent of other properties of observers. For example, the probabilities depend on the information that an observer has, and they are independent of the time of obtaining of this information.

The information about the card C_2 is a part of the information. The number of cards in the pack is the initial condition. However, it is possible that the observer O_3 does not know the number of cards in the pack. In this case the number of cards in the pack is the condition in conditional probabilities. In a general case we must specify all the information in conditional probabilities: initial and boundary conditions, a frame of reference, and so on. An absolute probability is the convention. This is the conditional probability relative to minimal common information for all considered observers or that relative to one fixed observer.

Let us discuss similar problems in quantum theory. Consider the description of a pack of cards by using the mathematical formalism of state vectors. The pack of cards has evident properties and this example exhibits some properties of the mathematical formalism that are not clear in the case of microscopic objects. Consider the probability amplitudes instead of the probabilities. Assume

$$
\psi = |P^{1/2}| \exp(i\phi) \tag{3.8}
$$

where the phase ϕ is arbitrary. The pack of cards is the macroscopic object and the probabilities do not depend on phases. The right basis vectors are

$$\cdots |Q\clubsuit\rangle = \begin{pmatrix} 0 \\ \cdots \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \qquad |K\clubsuit\rangle = \begin{pmatrix} 0 \\ \cdots \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \cdots \tag{3.9}$$

These are the eigenvectors of the denominations of the cards. Consider the state vector of the card C_1 for the observer O_1. The state vector of the card C_1 at the point of time T_1 is

$$|C_1 T_1\rangle = \begin{pmatrix} 51^{-1/2} \ \exp(i\phi) \\ 52^{-1/2} \ \exp(i\phi) \\ 52^{-1/2} \ \exp(i\phi) \\ \cdots \\ 52^{-1/2} \ \exp(i\phi) \end{pmatrix} \tag{3.10}$$

The state vector of the card C_1 at the point of time T_3 is

$$|C_1 T_3\rangle = \begin{pmatrix} 0 \\ 51^{-1/2} \ \exp(i\phi) \\ 51^{-1/2} \ \exp(i\phi) \\ 51^{-1/2} \ \exp(i\phi) \\ 51^{-1/2} \ \exp(i\phi) \\ \cdots \\ 51^{-1/2} \ \exp(i\phi) \end{pmatrix} \tag{3.11}$$

This is the projection of the state vector $|C_1 T_1\rangle$. However, the state vector of the card C_1 at the point of time T_3 is equal to $|C_1 T_1\rangle$ for the observer O_2. The state vector (3.11) is the state vector for the observer O_1 and not the state vector for the observer O_2. *In a general case the state vectors of the same object at the same point of time are different for different observers*.

The probabilities do not describe a single event. A single event takes place or does not take place. The probability of an event is the number of such events in some set of possible events divided by the number of all events in this set of possible events. If some observer obtains some information, the single event does not change. The observer only changes his own opinion about the set of possible events.

The measurement consists of two parts. The first is the interaction between an object and a device. This interaction can change the object and must be described by equations of motion. The second is the obtaining of new information by some observer. This change is described by a projection of a state vector, a reduction of a wavepacket in quantum theory, or by conditional probabilities in theory of probability. *We must use conditional amplitudes and conditional state vectors like conditional probabilities if we want to describe the points of view of different observers with different information in quantum theory*. Now only absolute state vectors are used in quantum theory. This description concerns the set of observers that have the same information and consequently the same set of possible events.

The macroscopic object has some properties that do not depend on observers. For example these are the denominations of cards. The description is more complicated for microscopic objects.

In the considered model of causal graphs the elementary objects are the edges and vertexes. These objects have no internal properties. All properties are their positions in the causal graph that are described by the structure of the causal graph. However, the observer's information is also the structure of the causal graph. The properties of edges and vertexes and the information about these properties are the same.

The objects of the next level are the subgraphs with the fixed structures that include some small numbers of edges and vertexes. The internal properties of such objects are their internal structures. The aim of the dynamics is to investigate the correlation between these subgraphs in the graph of the universe.

3.2. Correlations of Structures

We can make up the catalogue of all structures $S_a(e, v)$ of causal graphs that consist of e edges and v vertexes for any finite whole numbers e and v.

Consider the sufficiently large finite causal graph G_{universe}. For example G_{universe} is the known part of the universe. Consider all the partitions of G_{universe} into all possible causal graphs G_b. Each G_b is some structure $s_a(e, v)$

$$G_b = s_a(e, v) \tag{3.12}$$

Count the numbers $n(s_a(e, v))$ of $G_b = s_a(e, v)$, in all the partitions of G_{universe} for each structure $s_a(e, v)$. If the same G_b is included in different partitions, it is counted only once. Consider randomly chosen G_b with fixed e and v. By definition

$$P\{G_b = s_a(e, v) \mid G_{\text{universe}}\} = \left[\sum_{k=1}^{K} n(s_k(e, v))\right]^{-1} n(s_a(e, v)) \tag{3.13}$$

This is the probability that $G_b = s_a(e, v)$, where K is the number of different structures $s_a(e, v)$ that consist of e edges and v vertexes. The definition (3.13) is the definition of words: "... *randomly* chosen G_b with fixed e and v."

Suppose the probabilities (3.13) of the causal graphs in the earth's laboratory do not depend on the G_{universe} for sufficiently large G_{universe}. We will consider below the conditional probability (3.13) to be the absolute probability $P\{G_b = s_a(e, v)\}$.

Consider some history h_z of some object z. For example, this is the history of some lorry during a day. In the considered model h_z is the set of causal graphs. This set is a convention. For example, we can either include the history of the broken lorry in this set or not. Assume that by definition the history of any object includes only connected causal graphs.

Consider randomly chosen G_b. Using (3.13), we get

$$P\{G_b \in h_z\} = \sum_{s_k(e,v)\in h_z} P\{G_b = s_k(e, v)\} \tag{3.14}$$

This is the probability that a randomly chosen area of spacetime is a lorry. This is a very small value. Consider this probability if we know the structure of some connected causal graph G_1. If $G_b \cup G_1$ is a disconnected graph, the probability (3.14) does not change. The information that G_1 exists in the universe is already included in the definition (3.13). We sum up only the numbers of the existing causal graphs.

Assume that $G_b \cup G_1$ is a connected graph. If in general case the probability (3.14) does not change, this is the universe without correlations between events, this is chaos. In the real universe the events correlate and in a general case we get

$$P\{G_b \in h_z \mid G_1\} \neq P\{G_b \in h_z\} \tag{3.15}$$

For example, h_1 is the history of the lorry in the course of a year, $G_1 \in h_1$, and most of the vertexes of G_1 are the cause of most of the vertexes of G_b. In this case $P\{G_b \in h_z \mid G_1 \in h_1\}$ is the probability that the lorry will exist during the next day if it existed in the course of the previous year. This is not a very small value.

Consequently, only connected causal graphs are considered below.

3.3. A Growth of a Causal Graph

Now I introduce the following concept of a causal graph dynamics. Suppose we have all the information about the structure of some causal graph G. The aim of a causal graph dynamics is to calculate probabilities of the structures of other causal graphs G_x that are connected with G.

We can investigate the structure of the causal graph G_x step by step. This procedure is called the sequential growth dynamics for the causal graph G. Growth of the connected causal graph G is a sequence of some elementary processes. Such

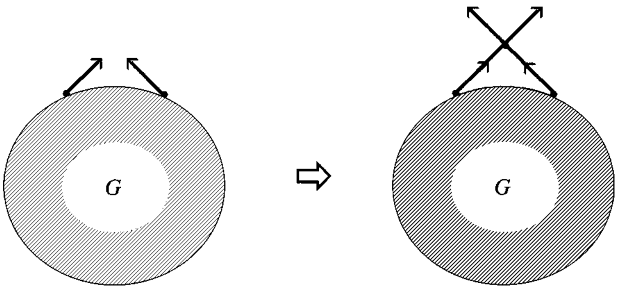

Fig. 4. The first type of the elementary interaction.

process is called an elementary interaction. Consider all types of the elementary interactions.

The first type is the generation of a new vertex by two external output edges (Fig. 4). The second type is a generation of a new vertex by two external input edges (Fig. 5). This growth of the causal graph in the past does not contradict to the causality. This is not an appearance of a new vertex. This is an appearance of new information about the past. The third type is a merging of one external input and one external output edge (Fig. 6). This merging can occur if the causality principle is not broken. The incident vertex of the external input edge is not the cause of the incident vertex of the external output edge. The fourth type is a generation of a new vertex by one external output edge and the edge that is not connected with the considered causal graph (Fig. 7). The fifth type is a generation of a new vertex by one external input edge and the edge that is not connected with the considered causal graph (Fig. 8). According to the fundamental law of conservation and the binary principle, the number of external edges does not change in the elementary interactions of the first and second types; the number of the external input and output edges decreases by one in the elementary interactions of the third type; the

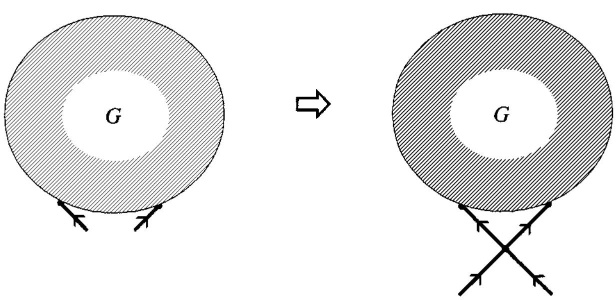

Fig. 5. The second type of the elementary interaction.

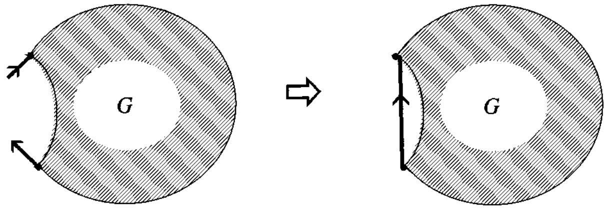

Fig. 6. The third type of the elementary interaction.

number of the external input and output edges increases by one in the elementary interactions of the fourth and fifth types. The elementary interactions of the first, second, and third types describe interactions of the edges of the causal graph. The elementary interactions of the fourth and fifth types describe interactions of the edge of the causal graph with all other universe.

All the variants of the growth of the connected causal graph can be divided into the elementary interactions of these five types. The sequence of elementary interactions is the sequence of the obtaining of the information about the structure of the causal graph by the observer. This sequence is not connected with the causal order of the causal graph.

The elementary interaction is the interaction of the external edges. The interaction of the vertexes is the interactions of the external edges incident to these vertexes. Consequently, the edge representation is preferable.

3.4. Probabilities of Elementary Interactions

The task is calculation of probabilities of any variants of the structure of the causal graph obtained from the given causal graph as a result of stochastic growth. Three procedures are necessary for the definition of this stochastic growth. The first procedure is the determination of the structure of the given causal graph. The second procedure is the determination of the probabilities of its elementary

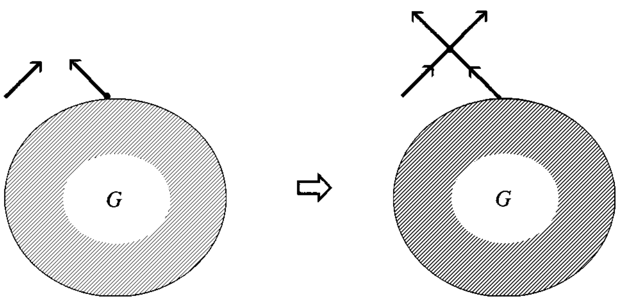

Fig. 7. The fourth type of the elementary interaction.

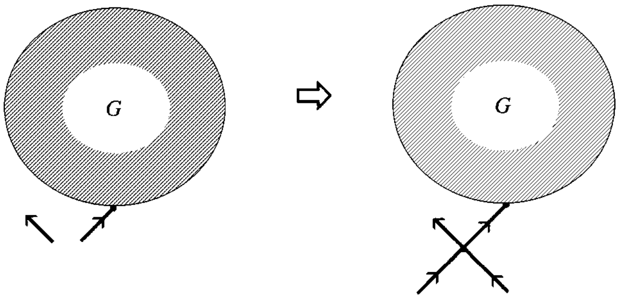

Fig. 8. The fifth type of the elementary interaction.

interactions. The third procedure is the calculation of probabilities of different sequences of elementary interactions.

It is meaningless to talk about the exact structure of the causal graph if we cannot determine its structure. Supposing the following procedure exists, we can stochastically initiate one elementary interaction for any given causal graph and we can determine the exact change of the structure of the causal graph that is the result of this elementary interaction. This procedure is called the elementary measurement.

We can construct a causal graph and determine the exact structure of this causal graph using elementary measurements. Consider the causal graph that consists of one isolated edge. Only the elementary interactions of the fourth and fifth types are possible for this causal graph. In this case these elementary interactions are the same and the probability of this elementary interaction is equal to one. We get the causal graph that is the X structure (Fig. 2). We get the causal graph with the known structure using the sequence of N elementary measurements.

Repeating the sequence of N elementary measurements n times, we get a set $\{G(N)\}$ of n causal graphs. This set consists of subsets of the causal graphs with the identical structure. Suppose the subset $\{G(N) = s_1(e, v)\}$ of causal graphs consists of k elements. Suppose the subset $\{G(N) = s_2(e, v)\}$ of causal graphs consists of l elements. By assumption, the ratio k/l tends to the real ratio of the numbers of causal graphs $G(N) = s_1(e, v)$ and $G(N) = s_2(e, v)$ in the graph of the universe if $n \to \infty$. This is the meaning of the words “We can *stochastically* initiate one elementary interaction . . .” in the definition of the elementary measurement.

Consider the set $\{G(N) = s_1(e, v)\}$ of the same causal graphs that are the result of the sequence of N elementary measurements. This set consists of k elements. Adding one elementary measurement, we get the set $\{G(N + 1)\}$ of the causal graphs that are the result of the sequence of $N + 1$ elementary measurements. This set consists of k elements. Consider the subset of the identical causal graphs that are the result of the elementary interaction of the external edges α and β of the causal graph $G(N) = s_1(e, v)$. Suppose this subset consists of r elements. Denote by $P\{\alpha, \beta\}$ the probability of the interaction of the external edges α and β. By definition,

$$P\{\alpha, \beta\} = \lim_{k\to\infty} \left(\frac{r}{k}\right) \tag{3.16}$$

We must write $P\{\alpha, \beta \mid G(N) = s_1(e, v)\}$ instead of $P\{\alpha, \beta\}$ in (3.16), but we shall omit the designation of the causal graph for simplification of the designation if the designation of the causal graph is evident. No external edge can interact with itself. Denoting the probability of the elementary interactions of the fourth and fifth types by $P\{\alpha\alpha\}$ and using the definition (3.16), we get

$$\sum_{\alpha}\sum_{\beta\leqslant\alpha} P\{\alpha, \beta\} = \lim_{k\to\infty} \left(\left(\sum_{\alpha}\sum_{\beta\leqslant\alpha} r\right) k^{-1}\right) = \lim_{k\to\infty} \left(\frac{k}{k}\right) = 1 \tag{3.17}$$

We get

$$P\{\alpha\} = \sum_{\beta} P\{\alpha, \beta\} \tag{3.18}$$

for the probability of the elementary interaction of the external edge α with any external edge.

$P\{\alpha, \beta\}$ is the classical probability and the considered model is a particular kind of the theory with hidden variables. If we can calculate the probabilities of all the elementary interactions of any causal graph, we can calculate the probabilities of all new parts of some causal graph as the probabilities of random sequences of elementary interactions. This is similar to the sequence of measurements in quantum theory. There is no interference of wavefunctions before and after the measurement. The sequence of measurements in quantum theory is the classical random sequence.

There are two dynamics. The first is the description of some process with the fixed information. This is a deterministic equation of motion. The second dynamics is the rules of addition of some new information. In this section the second dynamics of causal graphs is presented. If we get new information about the exact structure of some new part of the causal graph, all the probabilities are changed. The probability of this structure is equal to one. The probabilities of alternative structures are equal to zero. We must calculate other probabilities with the addition of this new information. This is an analogue of the quantum measurement and a projection of a state vector. Such dynamics is a “growth” of the causal graph. This is not an appearance of new parts of the causal graph. This is an appearance of new information about the existing causal graph.

The first dynamics of causal graphs are the deterministic equations for the calculation of $P\{\alpha, \beta\}$ for the fixed structure of the causal graph. These equations of motion can be very complicated. I assume that $P\{\alpha, \beta\}$ is the square of the modulus of some complex amplitude. This is the correspondence principle of the presented model and quantum theory. The description of causal graphs by complex amplitudes is presented in Section 5. The equations of the motion of causal graphs are the deterministic equations for the calculation of complex amplitudes. Some examples of such equations are presented in Krugly (1998, 2000a,b).

4. PHENOMENOLOGY OF CAUSAL GRAPH

4.1. A Time

The aim of phenomenology of causal graphs is to identify subgraphs and real objects and to discuss the physical meaning of some quantity in the model in question.

An oriented sequence is a discrete microscopic analogue of a continual macroscopic world line. By definition the microscopic time interval Δt_{ab} between two vertexes a and b of the oriented sequence S is the number k of the edges between a and b in S multiplied by some constant of time τ.

$$\Delta t_{ab} = \tau k \tag{4.1}$$

if a is the cause of b. Otherwise

$$\Delta t_{ab} = -\tau k \tag{4.2}$$

The time interval Δt_{ab} describes the causal order of the causal graph and is not connected with the time of observers.

Consider the oriented sequence S that is the result of the process of the sequential growth. Consider the causal graph $G(N_1)$. This graph is the result of N_1 elementary interactions. Consider some vertex a of $G(N_1)$ that is incident to some external output edge α_1. The growth of S from α_1 is the following process. We do not consider the elementary interactions of the third type for simplicity. α_1 does not take part in the first $n_1 - 1$ elementary interactions. α_1 interacts at the next elementary interaction (Fig. 9(a)). We get two external output edges. Choose one of these edges as α_2; α_2 does not interact at the next $n_2 - 1$ elementary interactions. α_2 interacts at the next elementary interaction (Fig. 9(b)). We get two external output edges. Choose one of these edges as α_3. After N elementary interactions we get the causal graph $G(N_1 + N)$ (Fig. 9(c)). In general case α_1 is the internal edge in $G(N_1 + N)$. We get some oriented sequence S from vertex a to some vertex b. S includes the internal edges $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_k$. The vertex b is incident to some external output edge α_{k+1}. Consider the same causal graphs $G(N_1)$ and repeat N elementary interactions. In general case we get different causal graphs $G(N_1 + N)$ and different oriented sequences S. Calculate the mean number k of the edges in S. By $P\{S \mid G(N_1 + n)\}$ denote the probability of the addition of one edge to S at the elementary interaction with the number $N_1 + n + 1$. This is

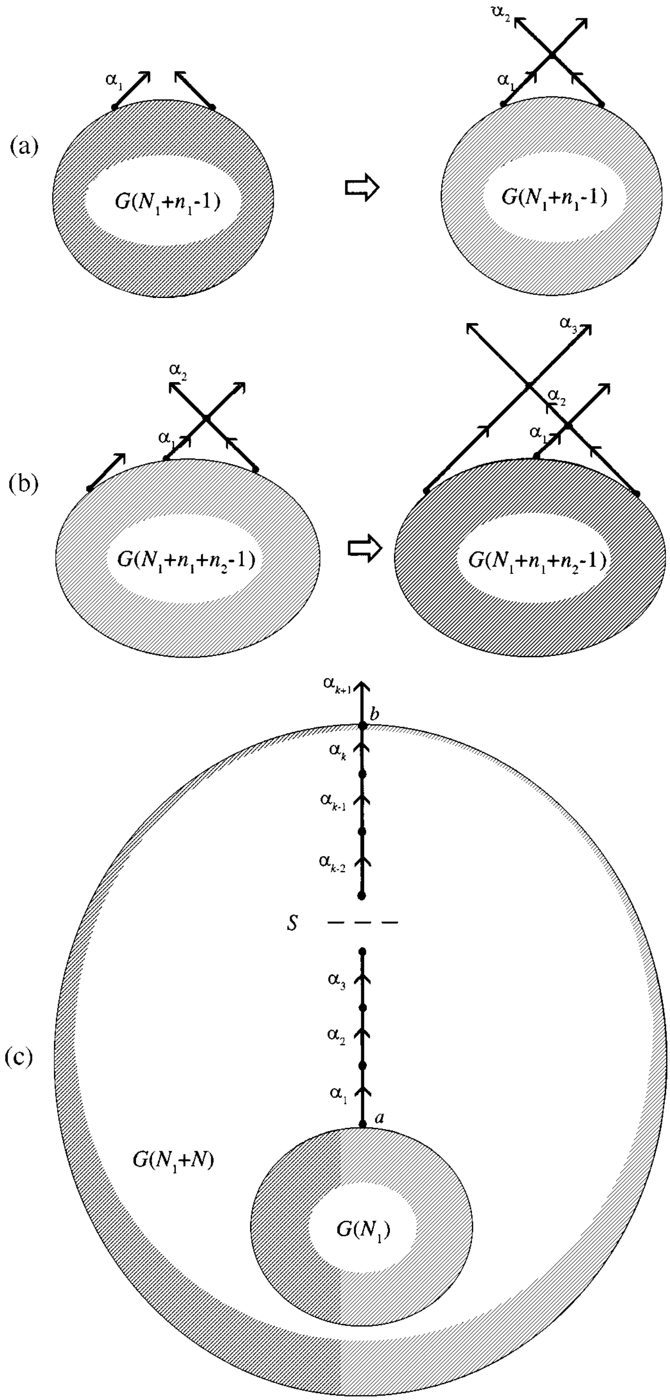

Fig. 9. The growth of the oriented sequence.

the probability (3.18) of the elementary interaction of the external output edge in the end of S. In a general case $P\{S \mid G(N_1 + n)\}$ depends on the results of the previous $N_1 + n$ elementary interactions. For the mean number of the edges in S after N elementary interactions we have

$$\langle k \rangle = P\{S \mid G(N_1)\} + P\{S \mid G(N_1 + 1)\} + \cdots + P\{S \mid G(N_1 + N - 1)\} \tag{4.3}$$

In a simple case all the probabilities $P\{S \mid G(N_1 + n)\}$ are equal. Then this probability of the elementary interaction of the external output edge α_i in the end of S is independent of the number i of α_i. Denoting this probability by $P\{\alpha\}$, in this case

$$\langle k \rangle = N P\{\alpha\} \tag{4.4}$$

The meaning of this equation is trivial. If the probability of the elementary interaction of the edges in the oriented sequence is greater, the interactions occur more often, and the oriented sequence has more edges. We get

$$\sigma^2(k) = N P\{\alpha\}(1 - P\{\alpha\}) \tag{4.5}$$

for the dispersion $\sigma(k)$ of k. Using (4.1) and (4.4), we get

$$\langle \Delta t_{ab} \rangle = \tau N P\{\alpha\} \tag{4.6}$$

for the mean microscopic time interval between two vertexes a and b. We get

$$\sigma^2(\Delta t_{ab}) = \tau N P\{\alpha\}(1 - P\{\alpha\}) \tag{4.7}$$

for the dispersion $\sigma(\Delta t_{ab})$ of Δt_{ab}.

Consider a “small” causal graph that corresponds to one infinitesimal macroscopic world line. The infinitesimal macroscopic proper time interval dT is uniquely defined in relativity theory using a standard point clock. We must define the standard point clock in the considered model in order to define the macroscopic proper time interval. A clock is some periodical process. In this model a point process is an oriented sequence. Let the standard point clock be some standard oriented sequence. By definition the macroscopic proper time interval ΔT is the microscopic time interval of the standard oriented sequence. We use ΔT instead of dT because there are no infinitesimal quantities in discrete models. Using (4.1), we get

$$\Delta T = \tau k_{\text{clock}} \tag{4.8}$$

for ΔT, where k_{clock} is the number of edges in the standard oriented sequence.

Consider the meaning of the standard oriented sequence. Let the standard oriented sequence be the result of the sequence of elementary interactions with a standard probability.

$$P\{\alpha\} = P\{\text{clock}\} \tag{4.9}$$

Let μ be the number of the external edges of G. $P\{\text{clock}\}$ is normalized by condition (3.17). Consequently, $P\{\alpha\}$ decreases if μ increases. Modifying the condition (3.17) to exclude the influence of μ, we have

$$\sum_{\alpha}\sum_{\beta\leqslant\alpha} P_\mu\{\alpha,\beta\} = \mu \tag{4.10}$$

$$P_\mu\{\alpha,\beta\} = \mu P\{\alpha,\beta\} \tag{4.11}$$

Consequently,

$$P_\mu\{\alpha\} = \mu P\{\alpha\} \tag{4.12}$$

Let the standard point clock be the oriented sequence with the constant probability

$$P_\mu\{\text{clock}\} = \text{const} \tag{4.13}$$

In this case the probability $P\{\text{clock}\}$ of the elementary interaction of the standard point clock depends only on μ.

$$P\{\text{clock}\} = \mu^{-1} P_\mu\{\text{clock}\} \tag{4.14}$$

Using (4.4), (4.8), (4.9), and (4.14), we get

$$\langle \Delta T\rangle = \tau P\{\text{clock}\} N = \tau P_\mu\{\text{clock}\}\mu^{-1} N \tag{4.15}$$

for the mean macroscopic proper time interval. Using (4.7), (4.9) and (4.14), we get

$$\begin{aligned}\sigma^2(\Delta T) &= \tau^2 P\{\text{clock}\} N(1 - P\{\text{clock}\})\\ &= \tau^2 P_\mu\{\text{clock}\}\mu^{-1} N(1 - P_\mu\{\text{clock}\}\mu^{-1})\end{aligned} \tag{4.16}$$

for the dispersion $\sigma(\Delta T)$ of ΔT. Equations (4.6) and (4.7) are valid in a simple case if $P\{\alpha\}$ is constant. Consequently, using (4.9), (4.13), and (4.14), we see that Eqs. (4.15) and (4.16) are valid if μ is constant. In this case only elementary interactions of the first and second types are allowable. Consequently,

$$P\{\alpha\alpha\} = 0 \tag{4.17}$$

for all external edges. Using (3.17), (3.18), (4.6), and (4.17), we get

$$\begin{aligned}\langle\langle \Delta t_{ab}\rangle\rangle_G &= \tau N\left(\mu^{-1}\sum_{\alpha} P\{\alpha\}\right) = \tau N\left(\mu^{-1}\sum_{\alpha}\sum_{\beta} P\{\alpha,\beta\}\right)\\ &= \tau N\left(2\mu^{-1}\sum_{\alpha}\sum_{\beta\leqslant\alpha} P\{\alpha,\beta\}\right) = 2\tau N\mu^{-1}\end{aligned} \tag{4.18}$$

for averaged $\langle \Delta t_{ab} \rangle$ of all oriented sequences in $G(N_1 + N)$. The right side of Eq. (4.15) has the clear physical meaning that ΔT is proportionate to $\langle\langle \Delta t_{ab} \rangle\rangle_G$.

For the causal graph corresponding to a set of macroscopic world lines, the definition of the macroscopic proper time interval is not discussed in this paper because in this case we must define continuous spacetime as a limit of the causal graph. The definition of the macroscopic proper time interval for one infinitesimal macroscopic world line can be the first step.

Time is a parameter in quantum theory. In this model time is a stochastic variable.

4.2. A Mass

Let the set of oriented sequences be corresponded to the same segment of the macroscopic world line. Each oriented sequence S is some physical process. We can describe this process using the cyclic frequency $\omega(S)$. By definition, put

$$\omega(S) = \varepsilon k (\Delta T)^{-1} \tag{4.19}$$

where ε is some constant of a phase, k is a number of the edges in S. ΔT is the macroscopic proper time interval of the considered segment of the macroscopic world line. In a general case, different oriented sequences have different k numbers of the edges and different cyclic frequences.

We have for the phase difference between the ends of S

$$\phi(S) = \omega(S)\Delta T = \varepsilon k \tag{4.20}$$

Consequently, ε is the phase difference between the ends of one edge. *All edges are identical and ε is a universal constant*. This is the main assumption. It is possible that ε is equal to $\pi/2$ (Krugly, 1998, 2000a,b); however, this value is not used in this paper.

Using (4.4), (4.12), (4.14), (4.15), and (4.19), we get

$$\begin{aligned} \langle \omega(S) \rangle &= \varepsilon \tau^{-1} P\{\text{clock}\}^{-1} P\{\alpha\} \\ &= \varepsilon \tau^{-1} P_\mu\{\text{clock}\}^{-1} P_\mu\{\alpha\} \end{aligned} \tag{4.21}$$

for the mean cyclic frequency; ε, τ and $P_\mu\{\text{clock}\}$ are universal constants and the mean cyclic frequency in (4.21) depends only on $P_\mu\{\alpha\}$. Equations (4.4), (4.15), and (4.21) are valid if $P_\mu\{\alpha\}$ is the constant for the growth of S. The meaning of Eq. (4.21) is trivial. If the probability of the elementary interaction of the edges in the oriented sequence is greater than such probability for the standard oriented sequence, the interactions occur more often than in the standard oriented sequence, and the considered oriented sequence has more edges.

The probability $P_\mu\{\alpha\}$ is the property of the single external edge α. Consequently, we can consider the cyclic frequency (4.21) as the cyclic frequency of the external edge α. By $\omega\{\alpha\}$ denote this cyclic frequency. By definition, put

$$\begin{aligned}\omega\{\alpha\} &= \varepsilon\tau^{-1}P\{\text{clock}\}^{-1}P\{\alpha\} \\ &= \varepsilon\tau^{-1}P_\mu\{\text{clock}\}^{-1}P_\mu\{\alpha\} \end{aligned} \tag{4.22}$$

The definition (4.22) is valid for the single elementary interaction. Assume this definition in general case. The simple case with the constant $P_\mu\{\alpha\}$ is only an illustration of the physical meaning of this definition.

$$\hbar\omega = mc^2 \tag{4.23}$$

for a particle in quantum theory, where $\hbar$ is the Planck's constant, m is the mass of a particle, and c is the speed of light. By definition, put

$$m\{\alpha\} = \hbar c^{-2}\omega\{\alpha\} \tag{4.24}$$

$m\{\alpha\}$ is called the mass of the external edge α. *In the considered model the external edges are particles*. In Section 5.6 the external edges are described by Dirac's equation.

Using (4.22) and (4.24), we get

$$m\{\alpha\} = m_0 P_\mu\{\alpha\} \tag{4.25}$$

for $m\{\alpha\}$, where

$$m_0 = \hbar c^{-2}\varepsilon\tau^{-1}P_\mu\{\text{clock}\}^{-1} \tag{4.26}$$

is the universal constant of mass. *The mass of the particle is the probability of the elementary interaction*. The proper time interval ΔT is used in the definition (4.19) of the cyclic frequency. Consequently, $m\{\alpha\}$ is the rest mass. If the minimum value $P_{\mu,\,\min} > 0$ of $P_\mu\{\alpha\}$ exists, $m_0 P_{\mu,\,\min}$ is the minimum mass of the particle. If the maximum value $P_{\mu,\,\max}$ of $P_\mu\{\alpha\}$ exists, $m_0 P_{\mu,\,\max}$ is the maximum mass of the particle.

$P_\mu\{\alpha\}$ describes the intensity of interactions of the external edge α. Therefore, the rest mass describes the intensity of interactions of a particle with all other universe. This is Mach's principle. Mach's principle states, generally speaking, that local inertia is determined by the overall distribution of the mass in the whole universe.

The case of external output edges has been considered above. Equation (4.25) defines the rest mass of the external output edge $\alpha\uparrow$.

$$m\{\alpha\uparrow\} = +m_0 P_\mu\{\alpha\uparrow\} = +m_0\mu P\{\alpha\uparrow\} \tag{4.27}$$

The consideration of the external input edges is the same, but we must use (4.2) instead of (4.1). Consequently, we must change the signs of $\langle\Delta t_{ab}\rangle$ in (4.6), ΔT in (4.8), $\langle\Delta T\rangle$ in (4.15), $\omega(S)$ in (4.19), $\phi(S)$ in (4.20), $\langle\omega(S)\rangle$ in (4.21), $\omega\{\alpha\}$ in

(4.22), and $m\{\alpha\}$ in (4.24) and (4.25). We get

$$m\{\alpha\downarrow\} = -m_0 P_\mu\{\alpha\downarrow\} = -m_0 \mu P\{\alpha\downarrow\} \qquad (4.28)$$

for the rest mass of the external input edge $\alpha\downarrow$. The external input edges are antiparticles. The external output edges become the external input edges if the orientation of the edges is inverted. This is the *CPT* reversal.

The model in question describes the asymmetry of the matter and the antimatter. The number of the external output edges is equal to the number of the external input edges in a causal graph according to the fundamental law of conservation. This is the symmetry between particles and antiparticles. However, the causal graph describes the present and the past. If we consider only the present there is the asymmetry between particles and antiparticles. Consider an external output edge. The rest of the external edges exist for this edge if they can interact with it. They are all external output edges and only a part of the external input edges. The interaction with the rest of the external input edges breaks the causality principle. These external input edges are in the past relative to the considered edge.

The elementary interaction of the third type is the annihilation (Fig. 6). The elementary interactions of the fourth and fifth types are the births of particle–antiparticle pairs (Figs. 7 and 8).

4.3. A Time–Energy Uncertainty Relation

Consider the measurement of the rest mass. We cannot measure $m\{\alpha\}$ or $\omega\{\alpha\}$ because the probability $P_\mu\{\alpha\}$ is not the directly measurable quantity. By assumption we can count only elementary interactions and edges in sequences.

Consider the following experiment. This is the growth of the oriented sequence presented in Section 4.1. We now consider the parallel growth of two oriented sequences (Fig. 10). We have the causal graph $G(N_1)$, which is the result of N_1 elementary interactions. We have the standard point clock as a part of the considered causal graph. Consider some external output edge α_1 of $G(N_1)$ that is incident to some vertex a. Let α_1 be the object under consideration. Consider some external output edge β_1 of $G(N_1)$ that is incident to some vertex c. Let β_1 be the standard point clock. After N elementary interactions we get the causal graph $G(N_1 + N)$. In a general case α_1 and β_1 are the internal edges in $G(N_1 + N)$. We get some oriented sequence S_1 from the vertex a to some vertex b. S_1 includes the internal edges $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_k$. The vertex b is incident to some external output edge α_{k+1}. We get some oriented sequence S_2 from the vertex c to some vertex d. S_2 includes the internal edges $\beta_1, \beta_2, \beta_3, \ldots \beta_j$. The vertex d is incident to some external output edge β_{j+1}. Consider the same causal graphs $G(N_1)$ and repeat N elementary interactions. In a general case we get different causal graphs $G(N_1 + N)$, different oriented sequences S_1, and different oriented sequences S_2. By assumption we can count edges in oriented sequences S_1 and S_2. Consequently,

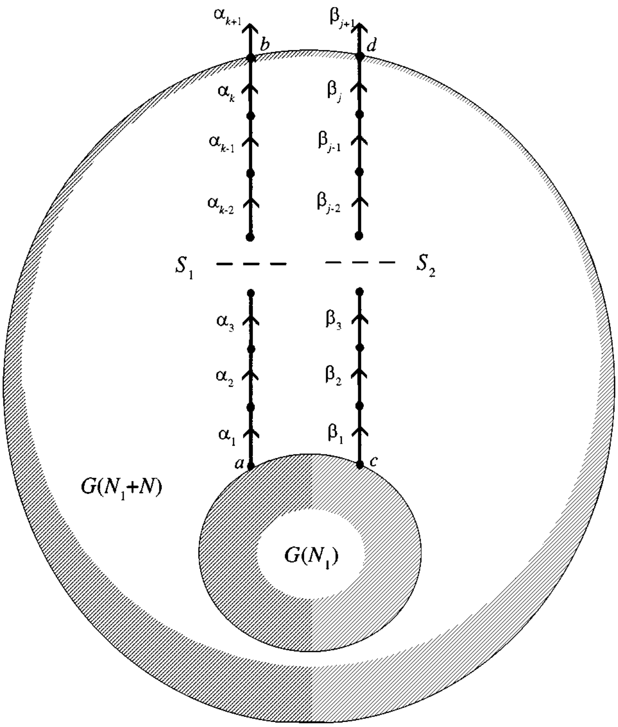

Fig. 10. The measurement of the rest mass.

using (4.8) and (4.19), we can determine ΔT and $\omega(S_1)$ for each causal graph $G(N_1 + N)$. By definition $\omega(S_1)$ is the measurable value of $\omega\{\alpha\}$. ΔT is the time of measurement. ΔT is not the time interval of the observer, ΔT is measured by the standard point clock of the causal graph.

Consider the dispersion of mass $\sigma(m\{\alpha\})$. We must express the mass as a function of measurable random quantities. Using (4.19) and (4.24), we get

$$m\{\alpha\} = \hbar c^{-2} \varepsilon k (\Delta T)^{-1} \tag{4.29}$$

If the random quantity x is the function of two random quantities y and z

$$x = f(y, z) \tag{4.30}$$

we have for the dispersion $\sigma(x)$

$$\sigma(x)^2 = \left(\frac{\partial f}{\partial y}\right)^2 \sigma(y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma(z)^2 \tag{4.31}$$

Using (4.4), (4.5), (4.15), (4.16), (4.29), and (4.31), we get

$$\sigma(m\{\alpha\})^2 = (\hbar c^{-2}\varepsilon\tau^{-1}P\{\text{clock}\}^{-1})^2 N^{-1} P\{\alpha\}((1 - P\{\alpha\}) + P\{\text{clock}\}^{-1}P\{\alpha\}(1 - P\{\text{clock}\})) \tag{4.32}$$

In quantum theory time is a parameter and does not have dispersion. Therefore a time interval is used in the uncertainty relation. In the considered model the standard point clock is a stochastic process. Hence the time interval is a random quantity with the dispersion (4.16) and we can use this dispersion in the uncertainty relation. Using (4.16) and (4.32), we get

$$\sigma(m\{\alpha\})\,\sigma(\Delta T) = \hbar c^{-2}\varepsilon\left(\frac{P\{\alpha\}}{P\{\text{clock}\}}\right)^{1/2}(1 - P\{\text{clock}\})^{1/2} \times \left(1 - P\{\alpha\} + \left(\frac{P\{\alpha\}}{P\{\text{clock}\}}\right)(1 - P\{\text{clock}\})\right)^{1/2} \tag{4.33}$$

The uncertainty relation (4.33) does not depend on N. Obviously, $P\{\alpha\} \leqslant 1$ and $P\{\text{clock}\} \leqslant 1$. If the minimum value $P_{\mu,\,\min} > 0$ of $P_\mu\{\alpha\}$ exists and μ is a big number in real experiments,

$$\begin{aligned}\sigma(m\{\alpha\})\,\sigma(\Delta T) &\geqslant \hbar c^{-2}\varepsilon\left(\frac{P_{\mu,\,\min}}{P_\mu\{\text{clock}\}}\right)(1 - P\{\text{clock}\})\\ &= \hbar c^{-2}\varepsilon\left(\frac{P_{\mu,\,\min}}{P_\mu\{\text{clock}\}}\right)(1 - \mu^{-1}P_\mu\{\text{clock}\})\\ &\cong \hbar c^{-2}\varepsilon\left(\frac{P_{\mu,\,\min}}{P_\mu\{\text{clock}\}}\right) = \text{const} > 0\end{aligned} \tag{4.34}$$

The classical probabilities of elementary interactions do not contradict to the uncertainty relation. Consequently, the model in question may be an unlocal theory with hidden variables.

5. AMPLITUDES OF ELEMENTARY INTERACTIONS

5.1. A State Vector of the Causal Graph

In this section, I introduce the description of the causal graph by the mathematical formalism of state vectors. Let us assume

$$P\{\alpha, \beta \mid G\} = \phi^*\{\alpha, \beta \mid G\}\phi\{\alpha, \beta \mid G\} \tag{5.1}$$

$\phi\{\alpha, \beta \mid G\}$ is called an amplitude of elementary interaction or an amplitude of the pair of the external edges α and β in G. The dynamics is complete if there is a law of calculation of $\phi\{\alpha, \beta\}$ for all the pairs of the external edges in any given causal graph. This is an equation of motion. I do not consider such a law in this paper and only assume that amplitudes exist and are unique for any pair of external edges and any causal graphs. There are some examples in Krugly (1998, 2000a,b). The amplitudes of the pairs of vertexes are discussed in these papers. These amplitudes may be the amplitudes of the pairs of the external edges incident to these vertexes. The aim of this section is to find a conformity between the properties of $\phi\{\alpha, \beta \mid G\}$ and the properties of quantum particles. All the equations in this section are descriptions of the fixed causal graph and we shall omit the designation of the causal graph for simplification of the designation.

It is evident that

$$P\{\alpha, \beta\} = P\{\beta, \alpha\} \tag{5.2}$$

Consequently,

$$|\phi\{\alpha, \beta\}| = |\phi\{\beta, \alpha\}| \tag{5.3}$$

Consider the state vector of the causal graph. The right vector is

$$|\phi\rangle = \begin{pmatrix} \phi\{1,1\} \\ \phi\{1,2\} \\ \cdots \\ \phi\{1,\mu\} \\ \phi\{2,1\} \\ \phi\{2,2\} \\ \cdots \\ \phi\{2,\mu\} \\ \cdots \\ \phi\{\alpha,1\} \\ \cdots \\ \phi\{\alpha,\beta\} \\ \phi\{\alpha,\beta+1\} \\ \cdots \\ \phi\{\alpha,\mu\} \\ \cdots \\ \phi\{\mu,1\} \\ \cdots \\ \phi\{\mu,\mu\} \end{pmatrix} \tag{5.4}$$

where μ is the number of the external edges. The dimension of the state vector of the causal graph is equal to μ^2. The component $\phi\{\alpha, \beta\}$ has the number $(\alpha - 1)\mu + \beta$. The left vector is

$$\langle\phi| = (\phi^*\{1, 1\} \cdots \phi^*\{\alpha, \beta\} \cdots \phi^*\{\mu, \mu\}) \tag{5.5}$$

The right basis vectors are

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \quad |n\rangle = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 1 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \quad |\mu^2\rangle = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 1 \end{pmatrix} \tag{5.6}$$

where the element with the number n of the column vector $|n\rangle$ is equal to one. The other elements of this vector are equal to zero. The left basis vectors are

$$\begin{aligned} \langle 1| &= (1\ \ 0\ \cdots\ 0\ \ 0\ \cdots\ 0) \\ \langle n| &= (0\ \ 0\ \cdots\ 1\ \ 0\ \cdots\ 0) \\ \langle \mu^2| &= (0\ \ 0\ \cdots\ 0\ \ 0\ \cdots\ 1) \end{aligned} \tag{5.7}$$

where the element with the number n of the row vector $\langle n|$ is equal to one. The other elements of this vector are equal to zero. Using (5.1)–(5.7), we get

$$P\{\alpha, \beta\} = \langle \phi \mid (\widehat{\alpha\beta}) \mid \phi \rangle = \langle \phi \mid (\alpha - 1)\mu + \beta \rangle \langle (\alpha - 1)\mu + \beta \mid \phi \rangle \tag{5.8}$$

for the probability of the elementary interaction. $(\widehat{\alpha\beta})$ is the operator of the elementary interaction of the external edges α and β. Consequently, the basis unit vectors (5.6) are the eigenvectors of the elementary interactions.

The state vector of the external edges is normalized by the condition

$$\sum_{\alpha=1}^{\mu} \sum_{\beta \leqslant \alpha} P\{\alpha, \beta\} = 1 \tag{5.9}$$

This representation is called the representation of the elementary interactions of the causal graph.

5.2. Total State Vectors of Edges

Consider amplitudes of the elementary interactions of the external edge α. These amplitudes form a total state vector of the external edge α.

$$|\phi\{\alpha\}\rangle = \begin{pmatrix} \phi\{\alpha, 1\} \\ \phi\{\alpha, 2\} \\ \cdots \\ \phi\{\alpha, \beta\} \\ \phi\{\alpha, \beta + 1\} \\ \cdots \\ \phi\{\alpha, \mu\} \end{pmatrix} \tag{5.10}$$

This representation is called the representation of the elementary interactions of the external edge α. The dimension of $|\phi\{\alpha\}\rangle$ is equal to μ. Let $\phi\{\alpha, \beta\}$ be the element $\phi_{\beta\alpha}$ of the square matrix Φ. This matrix is called the total amplitude matrix of the external edges. The total state vector $|\phi\{\alpha\}\rangle$ of the external edge α is the column of Φ with the number α. The left total state vector is

$$\langle\phi\{\alpha\}| = (\phi^*\{\alpha, 1\} \cdots \phi^*\{\alpha, \mu\}) \tag{5.11}$$

where $\phi^*\{\alpha, \beta\}$ is the element $\phi^*_{\alpha\beta}$ of the matrix Φ^+. The state vector $\langle\phi\{\alpha\}|$ of the external edge α is the row of Φ^+ with the number α. $|\phi\{\alpha\}\rangle$ is the vector in μ-dimensional space of states of the external edge α. The right basis vectors in this space are

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \quad |\beta\rangle = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 1 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \quad |\mu\rangle = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 1 \end{pmatrix} \tag{5.12}$$

where the element number β of the column vector $|\beta\rangle$ is equal to one. The other elements of this vector are equal to zero. The left basis vectors are

$$\begin{aligned} \langle 1| &= (1\ \ 0\ \cdots\ 0\ \ 0\ \cdots\ 0) \\ \langle\beta| &= (0\ \ 0\ \cdots\ 1\ \ 0\ \cdots\ 0) \\ \langle\mu| &= (0\ \ 0\ \cdots\ 0\ \ 0\ \cdots\ 1) \end{aligned} \tag{5.13}$$

where the element number β of the row vector $\langle\beta|$ is equal to one. The other elements of this vector are equal to zero. Using (5.1) and (5.10)–(5.13), we get

$$P\{\alpha, \beta\} = \langle\phi\{\alpha\} \mid \hat{\beta} \mid \phi\{\alpha\}\rangle = \langle\phi\{\alpha\} \mid \beta\rangle\langle\beta \mid \phi\{\alpha\}\rangle \tag{5.14}$$

for the probability of the elementary interaction; $\hat{\beta}$ is the operator of the elementary interaction of the external edges α and β in the representation of the elementary interactions of α. Consequently, the basis unit vectors (5.12) are the eigenvectors of the elementary interactions in this representation as the basis unit vectors (5.6) are the eigenvectors in the representation of the elementary interactions of the causal graph.

Using (3.18), (5.1), and (5.10), we get

$$P\{\alpha\} = \langle\phi\{\alpha\} \mid \phi\{\alpha\}\rangle \tag{5.15}$$

Consider the conditional probability $P\{\alpha, \beta \mid \alpha\}$. This is the probability of the elementary interaction of the external edges α and β if the probability of the elementary interaction of α with any edge is equal to one.

$$P\{\alpha, \beta \mid \alpha\} = P\{\alpha\}^{-1} P\{\alpha, \beta\} \tag{5.16}$$

Consider the normalized total state vector of the external edge

$$|\psi\{\alpha\}\rangle = |P\{\alpha\}^{-1/2}\,||\,\phi\{\alpha\}\rangle \tag{5.17}$$

The normalized left total state vector is

$$\langle\psi\{\alpha\}| = |P\{\alpha\}^{-1/2}|\langle\phi\{\alpha\}| \tag{5.18}$$

Using (5.14) and (5.16)–(5.18), we get

$$\begin{aligned} P\{\alpha, \beta \mid \alpha\} &= \langle\psi\{\alpha\} \mid \hat{\beta} \mid \psi\{\alpha\}\rangle \\ &= \langle\psi\{\alpha\} \mid \beta\rangle\langle\beta \mid \psi\{\alpha\}\rangle \end{aligned} \tag{5.19}$$

The normalized total state vectors $|\psi\{\alpha\}\rangle$ of the external edges are normalized by the condition

$$\sum_{\beta=1}^{\mu} P\{\alpha, \beta \mid \alpha\} = \langle\psi\{\alpha\} \mid \psi\{\alpha\}\rangle = 1 \tag{5.20}$$

The elements $\psi\{\alpha, \beta\}$ of the matrix vectors $|\psi\{\alpha\}\rangle$ form the normalized total amplitude matrix Ψ of the external edges. The normalized total state vector $|\psi\{\alpha\}\rangle$ of the external edge α is the column of Ψ with the number α. The normalized total state vector $\langle\psi\{\alpha\}|$ of the external edge α is the row of Ψ^+ with the number α.

It is evident that in a general case

$$|\psi\{\alpha, \beta\}| \neq |\psi\{\beta, \alpha\}| \tag{5.21}$$

5.3. An Equation of Particles

The external edge α is described by the real scalar $m\{\alpha\}$ and the complex vector $|\psi\{\alpha\}\rangle$. The edges with different $m\{\alpha\}$ or $|\psi\{\alpha\}\rangle$ are different particles or different states of the same particle.

Let the set $\{m\{\alpha\}\}$ of G be the spectrum of the square matrix M, and the set $\{|\psi\{\alpha\}\rangle\}$ of G be the eigenvectors of M.

$$M|\psi\{\alpha\}\rangle = m\{\alpha\}|\psi\{\alpha\}\rangle \tag{5.22}$$

Equation (5.22) is called the equation of particles.

Let us find M. The number of the external edges of G is μ. The number of $m\{\alpha\}$ is μ. The number of $|\psi\{\alpha\}\rangle$ is μ. The order of M is μ. Let the vectors $|\psi\{\alpha\}\rangle$ be linear-independent. Consider the eigenspace of M. Let the vectors $|\psi\{\alpha\}\rangle$ be the new basis vectors. M is the diagonal matrix M_{diag} in this representation.

$$M_{\text{diag}} = \begin{pmatrix} m\{1\} & & 0 & \\ & m\{2\} & & \\ & & \dots & \\ 0 & & & m\{\mu\} \end{pmatrix} \tag{5.23}$$

$$M = \Psi M_{\text{diag}} \Psi^{-1} \tag{5.24}$$

Ψ is the unitary matrix if vectors $|\psi\{\alpha\}\rangle$ are orthogonal. M is the Hermitian matrix in this case. Using (5.17) and (5.18), we get

$$\Phi = \Psi P_{\text{diag}} \qquad \Phi^{-1} = P_{\text{diag}}^{-1} \Psi^{-1} \tag{5.25}$$

where

$$P_{\text{diag}} = \begin{pmatrix} |P\{1\}^{-1/2}| & & 0 \\ & |P\{2\}^{-1/2}| & \\ & \cdots & \\ 0 & & |P\{\mu\}^{-1/2}| \end{pmatrix} \tag{5.26}$$

The diagonal matrices are permutable matrices. Consequently,

$$M = \Phi M_{\text{diag}} \Phi^{-1} \tag{5.27}$$

5.4. State Vectors of Edges

The aim of this paper is to find a connection between the properties of the particles and the quantities that describe the causal graphs. The description of external edges by total state vectors looks like the description of quantum particles. But the dimension of $|\psi\{\alpha\}\rangle$ is equal to μ.

The vector $|\psi\{\alpha\}\rangle$ describes the elementary interactions of the external edge α with all other external edges. We consider only the elementary interactions of ν external edges of G and $\nu < \mu$ and number these edges from 1 to ν. Let $\alpha \leqslant \nu$ and $\beta \leqslant \nu$ below. All the equations in this case are similar to the previous equations if we replace μ by ν. If we consider only ν external edges, we must consider the conditional probability $P\{\alpha, \beta \mid (\nu)\}$. $P\{\alpha, \beta \mid (\nu)\}$ is the probability of the elementary interaction of the external edges α and β if only the elementary interactions of the considered external edges occur. This probability is normalized by the condition

$$\sum_{\alpha=1}^{\nu} \sum_{\beta \leqslant \alpha} P\{\alpha, \beta \mid (\nu)\} = 1 \tag{5.28}$$

Using (5.28), we get

$$P\{\alpha, \beta \mid (\nu)\} = \left(\sum_{\alpha=1}^{\nu} \sum_{\beta \leqslant \alpha} P\{\alpha, \beta\} \right)^{-1} P\{\alpha, \beta\} \tag{5.29}$$

Considering the amplitudes $\phi\{\alpha, \beta \mid (\nu)\}$, by definition

$$\phi\{\alpha, \beta \mid (\nu)\} = \left(\sum_{\alpha=1}^{\nu} \sum_{\beta \leqslant \alpha} P\{\alpha, \beta\} \right)^{-1/2} \phi\{\alpha, \beta\} \tag{5.30}$$

Using (5.1), (5.29), and (5.30), we get

$$P\{\alpha, \beta \mid (\nu)\} = \phi^*\{\alpha, \beta \mid (\nu)\}\phi\{\alpha, \beta \mid (\nu)\} \tag{5.31}$$

The amplitudes $\phi\{\alpha, \beta \mid (\nu)\}$ of the elementary interactions of the edge α form the state vector $|\phi\{\alpha \mid (\nu)\}\rangle$. The dimension of $|\phi\{\alpha \mid (\nu)\}\rangle$ is equal to ν. These amplitudes form the square matrix $\Phi(\nu)$. This matrix is called the amplitude matrix of the external edges. The state vector $|\phi\{\alpha \mid (\nu)\}\rangle$ of the external edges α is the column of $\Phi(\nu)$ with the number α. $|\phi\{\alpha \mid (\nu)\}\rangle$ is the vector in the ν-dimensional space of states of the external edge α. The right basis vectors in this space are

$$|1(\nu)\rangle = \begin{pmatrix} 1 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \quad |\beta(\nu)\rangle = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 1 \\ 0 \\ \cdots \\ 0 \end{pmatrix} \quad |\nu(\nu)\rangle = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 1 \end{pmatrix} \tag{5.32}$$

where the element number β of the column vector $|\beta(\nu)\rangle$ is equal to one. The other elements of this vector are equal to zero. Using (5.31) and (5.32), we get

$$\begin{aligned} P\{\alpha, \beta \mid (\nu)\} &= \langle\phi\{\alpha \mid (\nu)\} \mid \hat{\beta}(\nu) \mid \phi\{\alpha \mid (\nu)\}\rangle \\ &= \langle\phi\{\alpha \mid (\nu)\} \mid \beta(\nu)\rangle\langle\beta(\nu) \mid \phi\{\alpha \mid (\nu)\}\rangle \end{aligned} \tag{5.33}$$

for the probability of the elementary interaction. $\hat{\beta}(\nu)$ is the operator of the elementary interaction of the external edges α and β. Consequently, the basis unit vectors (5.32) are the eigenvectors of the elementary interactions in the ν-dimensional space of states of the external edge α. Similar to (5.15), we get

$$P\{\alpha \mid (\nu)\} = \langle\phi\{\alpha \mid (\nu)\} \mid \phi\{\alpha \mid (\nu)\}\rangle \tag{5.34}$$

for the probability of the elementary interaction of the external edges α with any other external edge $\beta \leqslant \nu$.

Normalizing the state vectors,

$$|\psi\{\alpha \mid (\nu)\}\rangle = |P\{\alpha \mid (\nu)\}^{-1/2} \mid \phi\{\alpha \mid (\nu)\rangle \tag{5.35}$$

The normalized amplitudes $\psi\{\alpha, \beta \mid (\nu)\}$ from the normalized amplitude matrix $\Psi(\nu)$ of the external edges. The state vector $|\psi\{\alpha \mid (\nu)\}\rangle$ of the external edge α is the column of $\Psi(\nu)$ with the number α.

Consider the conditional probability $P\{\alpha, \beta \mid \alpha(\nu)\}$. This is the probability of the elementary interaction of the external edges α and β if the probability of the elementary interaction of α with any external edge $\beta \leqslant \nu$ is equal to one.

$$\begin{aligned} P\{\alpha, \beta \mid \alpha(\nu)\} &= P\{\alpha \mid (\nu)\}^{-1}\ P\{\alpha, \beta \mid (\nu)\} \\ &= P\{\alpha \mid (\nu)\}^{-1}\langle\phi\{\alpha \mid (\nu)\} \mid \hat{\beta}(\nu) \mid \phi\{\alpha \mid (\nu)\}\rangle \\ &= \langle\psi\{\alpha \mid (\nu)\} \mid \hat{\beta}(\nu) \mid \psi\{\alpha \mid (\nu)\}\rangle \\ &= \langle\psi\{\alpha \mid (\nu)\} \mid \beta(\nu)\rangle\langle\beta(\nu) \mid \psi\{\alpha \mid (\nu)\}\rangle \end{aligned} \tag{5.36}$$

If we consider only the elementary interactions of the external edges $\alpha \leqslant \nu$ and $\beta \leqslant \nu$, we can consider the approximate mass. Replace $P\{\alpha\uparrow\}$ with $P\{\alpha\uparrow \mid (\nu)\}$ in (4.27). By definition put

$$m\{\alpha\uparrow \mid (\nu)\} = +m_0\nu P\{\alpha\uparrow \mid (\nu)\} \tag{5.37}$$

for the approximate rest mass of the external output edge $\alpha\uparrow$. Similarly

$$m\{\alpha\downarrow \mid (\nu)\} = -m_0\nu P\{\alpha\downarrow \mid (\nu)\} \tag{5.38}$$

for the approximate rest mass of the external input edge $\alpha\downarrow$.

If $\{|\psi\{\alpha \mid (\nu)\}\rangle\}$ are linear-independent, $\{|\psi\{\alpha \mid (\nu)\}\rangle\}$ are the eigenvectors and $\{m\{\alpha \mid (\nu)\}\}$ are the eigenvalues of the equation of particles

$$M(\nu)|\psi\{\alpha \mid (\nu)\}\rangle = m\{\alpha \mid (\nu)\}|\psi\{\alpha \mid (\nu)\}\rangle \tag{5.39}$$

where

$$M(\nu) = \Psi(\nu)M_{\text{diag}}(\nu)\Psi(\nu)^{-1} = \Phi(\nu)M_{\text{diag}}(\nu)\Phi(\nu)^{-1} \tag{5.40}$$

and

$$M_{\text{diag}}(\nu) = \begin{pmatrix} m\{1 \mid (\nu)\} & & 0 \\ & m\{2 \mid (\nu)\} & \\ & \cdots & \\ 0 & & m\{\nu \mid (\nu)\} \end{pmatrix} \tag{5.41}$$

Suppose we can determine some regular structure in different causal graphs and this structure consists of ν external edges. Consequently, this structure is described by the set of ν state vectors $\{|\psi\{\alpha \mid (\nu)\}\rangle\}$ and the set of ν approximate rest masses; ν is independent of the other parts of the causal graph. This description is similar to the description of the set of ν quantum particles.

5.5. Amplitudes of Internal Edges

The quantum particle is described at some instant of time by a set of amplitudes. If a measurement occurs, the amplitudes determine probabilities of different results of this measurement. However, without measurements the probabilities are meaningless. The amplitude is only some mathematical description at the instant of time without measurements. If we consider the amplitudes of all pairs of all edges, we get a similar description for the causal graph. We suppose that the amplitudes of all pairs of all edges of any causal graph exist and the structure of the causal graph uniquely determines these amplitudes. If both edges are external, the square of the modulus of the amplitude is the probability of the elementary interaction of these edges. If one edge or both edges are internal, the amplitude is only some mathematical description of the structure of the causal graph. The same edge is an internal edge for some observers and an external edge for other observers. The same mathematical description of external and internal edges allows a comparison of the properities of the same edge for different observers.

Only the external edges can interact. In this case we must use the following equation

$$P\{\alpha, \beta\} = \hat{e}\,\phi^*\{\alpha, \beta\}\phi\{\alpha, \beta\} \tag{5.42}$$

instead of Eq. (5.1). $\hat{e}$ is the operator of the external edges. By definition $\hat{e} = 1$ if α and β are external edges. Otherwise $\hat{e} = 0$. We must change all the equations where amplitudes determine probabilities. We must use the following equation

$$P\{\alpha, \beta\} = \langle \phi\{\alpha\} \mid \hat{e}\,\hat{\beta} \mid \phi\{\alpha\}\rangle \tag{5.43}$$

instead of Eq. (5.14). In this case the dimension of $|\phi\{\alpha\}\rangle$ is equal to the number of all the edges in the causal graph. There are similar changes in the other equations.

The third type of elementary interaction is a merging of one external input edge and one external output edge (Fig. 6). This merging can occur if the causality principle is not broken. The incident vertex of the external input edge is not the cause of the incident vertex of the external output edge. It is useful to describe this prohibition as a selection rule. Suppose the amplitudes are not equal to zero in a general case for such interaction. In this case we must use the following equation

$$P\{\alpha, \beta\} = \hat{c}\,\hat{e}\,\phi^*\{\alpha, \beta\}\phi\{\alpha, \beta\} \tag{5.44}$$

instead of Eq. (5.42). $\hat{c}$ is the causal operator. By definition $\hat{c} = 0$ for the elementary interaction of the third type if the incident vertex a of the external input edge α is the cause of the incident vertex b of the external output edge β. Otherwise $\hat{c} = 1$; $\hat{c} = 1$ for the elementary interactions of the first, second, fourth, and fifth types. We must change all equations where amplitudes determine probabilities. We must use the following equation

$$P\{\alpha, \beta\} = \langle \phi\{\alpha\} \mid \hat{c}\,\hat{e}\,\hat{\beta} \mid \phi\{\alpha\}\rangle \tag{5.45}$$

instead of Eq. (5.43). Evidently, the operators $\hat{e}$ and $\hat{c}$ commute.

There is a simple algorithm to calculate $\hat{c}$ for the elementary interactions of the third type. Consider the vertex incidence matrix $V(G)$. The matrix element v_{ab} is equal to 1 if there is an oriented edge between the initial vertex a and the final vertex b. Otherwise $v_{ab} = 0$. The element $v_{ab}(n)$ of the matrix $(V(G))^n$ is equal to the number of all the oriented sequences with n edges between the initial vertex a and the final vertex b. Let r be the number of the internal edges of G. Let k be the maximal number of the internal edges of G that are included in the same oriented sequence. It is clear that $k \leqslant r$. Consequently, $v_{ab}(n) \equiv 0$ if $r < n$. Let

$$\hat{c} = c\left(\sum_{n=0}^{r} v_{ab}(n)\right) \tag{5.46}$$

for the incident vertex a of the external input edge and the incident vertex b of the external output edge. $c(x) = 1$ if $x = 0$. Otherwise $c(x) = 0$.

5.6. Dirac's Equation of Free Particles

Consider the base structures of the causal graph. The simplest structure is an edge. This is a trivial structure. The next structure is a vertex. Each vertex has four incident edges: two input edges and two output edges (Fig. 2). This is the consequence of the fundamental law of conservation and the binary principle. This structure was called X structure in Section 2.3. In this case $v = 4$. Let

$$\begin{aligned}\langle\phi^*\{1 \mid (4)\} \mid \phi\{1 \mid (4)\}\rangle &= \langle\phi^*\{2 \mid (4)\} \mid \phi\{2 \mid (4)\}\rangle \\ &= \langle\phi^*\{3 \mid (4)\} \mid \phi\{3 \mid (4)\}\rangle \\ &= \langle\phi^*\{4 \mid (4)\} \mid \phi\{4 \mid (4)\}\rangle \\ &= (4m_0)^{-1}m \end{aligned}\tag{5.47}$$

where m is some real positive number. If these edges are external edges, using (5.34) and (5.47), we get

$$P\{1 \mid (4)\} = P\{2 \mid (4)\} = P\{3 \mid (4)\} = P\{4 \mid (4)\} = (4m_0)^{-1}m \tag{5.48}$$

Then

$$M_{\text{diag}}(4) = \begin{pmatrix} m & & 0 & \\ & m & & \\ & & -m & \\ & 0 & & -m \end{pmatrix} \tag{5.49}$$

Using (5.39), (5.40), and (5.49), we get

$$M(4)|\psi\{\alpha \mid (4)\}\rangle = \pm m|\psi\{\alpha \mid (4)\}\rangle \tag{5.50}$$

where

$$M(4) = \Psi(4)M_{\text{diag}}(4)\Psi(4)^{-1} = \Phi(4)M_{\text{diag}}(4)\Phi(4)^{-1} \tag{5.51}$$

Equation (5.50) coincides with Dirac's equation in the momentum representation. This is the momentum representation because all the numbers in (5.50) are c numbers. This is coordinated with the assumption in Section 2.2 that the edge representation is the momentum representation.

Consider a simple case that $\Phi(4) = \Phi_1(4)$.

$$\Phi_1(4) = \begin{pmatrix} w(p_0+m) & 0 & wp_3 & w(p_1-ip_2) \\ 0 & w(p_0+m) & w(p_1+ip_2) & -wp_3 \\ wp_3 & w(p_1-ip_2) & w(p_0+m) & 0 \\ w(p_1+ip_2) & -wp_3 & 0 & w(p_0+m) \end{pmatrix} \tag{5.52}$$

where

$$w = (2p_0)^{-1/2}(p_0+m)^{-1/2}m^{1/2} \tag{5.53}$$

p_0, p_1, p_2, and p_3 are the real numbers, and

$$p_0^2 - p_1^2 - p_2^2 - p_3^2 = m^2 \tag{5.54}$$

In this case Eq. (5.50) coincides with Dirac's equation of free particles in the standard representation. The equation (5.50) with $M_{\text{diag}}(4)$ coincides with Dirac's equation of particles at rest. Using (5.52), we get

$$\Phi_1(4) = m^{1/2}B \tag{5.55}$$

where B is the boost.

$$\det B = 1 \tag{5.56}$$

The columns of $\Psi_1(4) = B$ coincide with the state vectors $|\Psi\{\alpha(4)\}\rangle$ of particles with the spin 1/2.

In the considered case the edges are fermions that satisfy Pauli's exclusion principle. In general case the equation (5.50) exists if the state vectors of the edges are linear-independent columns of $\Psi(4)$. This is the form of Pauli's exclusion principle.

If the edges in X structure are external edges, we can calculate the range of the mass. Using the condition (5.28), we get

$$\sum_{\alpha=1}^{4}\sum_{\beta\leqslant\alpha} P\{\alpha, \beta \mid (4)\} = 1. \tag{5.57}$$

Using (5.48) and (5.57), we get

$$\begin{aligned}(4m_0)^{-1}m = \frac{1}{2} - \frac{1}{4}(&P\{1, 1 \mid (4)\} + P\{2, 2 \mid (4)\} \\ &+ P\{3, 3 \mid (4)\} + P\{4, 4 \mid (4)\})\end{aligned} \tag{5.58}$$

Obviously

$$0 \leqslant (P\{1, 1 \mid (4)\} + P\{2, 2 \mid (4)\} + P\{3, 3 \mid (4)\} + P\{4, 4 \mid (4)\})$$
$$\leqslant \sum_{\alpha=1}^{4} \sum_{\beta \leqslant \alpha} P\{\alpha, \beta \mid (4)\} = 1 \tag{5.59}$$

Using (5.58) and (5.59), we get

$$m_0 \leqslant m \leqslant 2m_0 \tag{5.60}$$

The consideration of the simple X structure does not provide the real value of the mass of the particles.

6. DISCUSSION

Suppose that state vectors of the edges are linear-independent and satisfy the condition (5.47) for each X structure in any causal graph. In this case all the edges are fermions with the spin $\frac{1}{2}$. They are building blocks of the universe. These conditions must be the consequences of the equation of motion that is the rule for calculation of the amplitudes. This is the prompting to searching of this equation.

The edges in X structure are described by Eq. (5.50). This equation is the relation between the fixed sets of numbers if the causal graph is fixed. Only the numbering of the edges is arbitrary. If we fix the causal graph, we fix the information about the causal graph. In other words we fix the observer. The fixed observer describes any object as a set of fixed numbers. Suppose the observer O_1 knows the causal graph G_1 and the observer O_2 knows the causal graph G_2. Both observers can describe the X structure X_1 if $X_1 \subset G_1 \cap G_2$. In general case the observers O_1 and O_2 describe the same X structure X_1 by the different equations (5.50) because the causal graphs G_1 and G_2 are different. The consequence of the change of the observer is a transformation of Eq. (5.50) and state vectors. In particular case this transformation is Lorentz's transformation. The choice of the causal graph is the discrete analogue of the choice of system of reference.

The set of possible causal graphs is finite if the graph of the universe is finite, or the set of possible causal graphs is a countable set if the graph of the universe is infinite. Consequently, all the sets of quantities that describe the causal graphs are finite or countable sets. In a particular case the state vectors form a finite or a countable set.

All the edges in X structures are described as the particles with the mass $\approx m_0$. Perhaps we can describe the edges as different particles with the real mass if we consider the more complicated structures that include a few neighbor vertexes. Such structures can describe the multiplets of particles.

The edge is a particle at the fixed point of time. If we want to describe the influence of the force fields, we must consider the particle at different points of time.

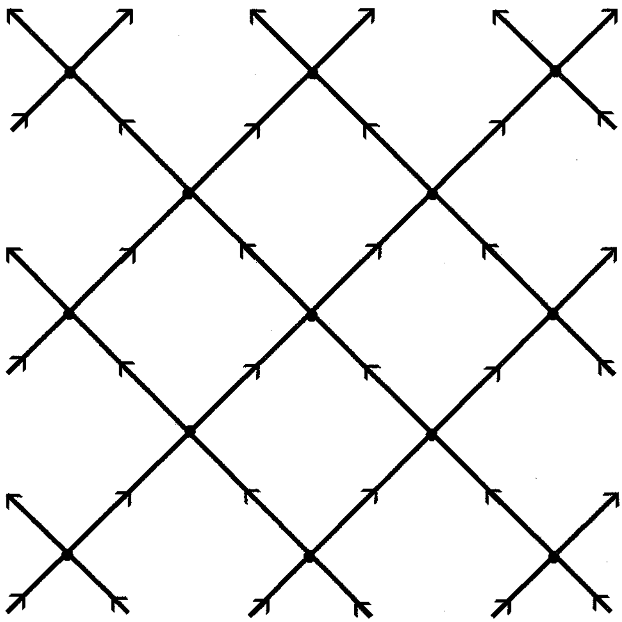

Fig. 11. The finite part of the infinite graph with the structure of a chessboard.

Consider the oriented sequence as the discrete world line of the particle. The two consecutive edges of this sequence are two consecutive states of the particle. If these states are described by the different states vectors, we can describe this difference as the influence of the force fields. An example of the absolutely symmetric graph is the infinite chessboard (Fig. 11). In this case all the X structures are described by the same Dirac's equation. It is possible that the quantums of force fields are the elementary breakings of the symmetry.

There are following ways for the future investigation of the considered model.

The first way is to investigate different structures that include a few neighbor vertexes. This is the investigation of the appropriate equation (5.39), the state vectors, their transformations and possible interpretation as the multiplets of particles, Lorentz's transformations, and gauge transformations.

The second way is to search the law of calculation of the amplitudes for the given causal graph. It is possible that this is some sum over sequences as the discrete analogue of the path integral.

The third way is to investigate the proceeding to the limit of continual spacetime. There are four parameters p_0, p_1, p_2, and p_3 in the amplitude matrix (5.52) that must be interpreted in a continual limit as components of the four-momentum. It is possible that we will be able to get spacetime by two steps. The first step provides us with the four-dimensional momentum space in each vertex. The second step provides us with spacetime by some kind of Fourier transformation. In this case the dimension $3 + 1$ of spacetime is the consequence of Eq. (5.50). We can call this dimension the dynamical dimension.

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